Horizontal atmospheric pressure gradients associated with condensation of water vapor

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Abstract. Condensation of water vapor in adiabatically ascending air produces a non-equilibrium vertical gradient of water vapor pressure. Here we show, based on an analysis of the continuity equation for a compressible mixture of condensable and non-condensable gas components, that the loss of mass of the condensable gas (water vapor) through condensation results in the formation of a horizontal gradient of total air pressure. The magnitude of this gradient is roughly proportional to the vertical non-equilibrium pressure gradient of the condensable component multiplied by the ratio of vertical to horizontal velocity. For large scale circulation features our estimated condensation-induced horizontal pressure gradient appears close to observations, at around 0.6 Pa km$^{-1}$ for Hadley cells. We conclude that condensation-

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induced dynamics merits further attention as a driver of atmospheric mo-
tions on Earth.
1. Introduction

Various authors have noted that precipitation reduces local air pressure [Lorenz, 1967; Trenberth et al., 1987; Trenberth, 1991; Gu and Qian, 1991; Ooyama, 2001; Schubert et al., 2001; Qiu et al., 1993; Lackmann and Yablonsky, 2004]. This makes sense based on the recognition that air pressure is equal to the weight of gas in the atmospheric column and that removal of substance via precipitation reduces pressure in the air column. However, no comprehensive attempt has been made to describe the spatial pressure changes associated with condensation from basic principles. Here we perform such an analysis to show that condensation of water vapor produces horizontal pressure gradients of observable magnitudes and discuss their relevance for the atmospheric dynamics on Earth.

We consider a moist saturated atmosphere as a mixture of a non-condensable component (dry air) with molar density $N_d$ and a condensable component (water vapor) with saturated molar density $N_v$. Molar density $N$ of moist air is thus $N = N_d + N_v$. The ideal gas equation of state relates pressure $p$ to temperature $T$ and molar density $N$ via the universal molar gas constant $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$ independently of molar mass:

$$p = NRT, \quad p_v = N_vRT, \quad p_d = N_dRT.$$  \hspace{1cm} (1)

Here $p$, $p_v$ and $p_d$ are the pressure of moist air as a whole, saturated water vapor and dry air, respectively.

To provide conceptual transparency we consider the case of a stationary flow, where the air moves horizontally along the $x$ axis and vertically along the $z$ axis; there is no dependence of the flow on the $y$ coordinate. The continuity equation for a gas mixture,
where some component (here water vapor) is not conserved, then reads:

\[
\frac{\partial (Nu)}{\partial x} + \frac{\partial (Nw)}{\partial z} = S(x, z).
\] (2)

Here \(u\) is horizontal velocity, \(w\) is vertical velocity and \(S(x, z)\) is the volume-specific rate of "loss" (condensation) of the condensable component (mol m\(^{-3}\) s\(^{-1}\)).

In the atmosphere the sink term \(S\) is equal to the difference between volume-specific evaporation rate and condensation rate, \(S = \tilde{E} - \tilde{C}\). Local recycling (evaporation – condensation – re-evaporation) does not change the amount of vapor in the local volume, but any imbalance in the rates does. Therefore, the mean column value of \(S\) can be estimated from the regional evaporation minus precipitation data, \(S = (E - P)/h_P\), where \(h_P\) is the characteristic height of the atmospheric layer harboring the bulk of the condensation process, \(E\) and \(P\) are the surface-specific values of evaporation and precipitation at the surface (mol m\(^{-2}\) s\(^{-1}\)).

2. Derivation of the Horizontal Pressure Gradient

As the moist saturated air ascends adiabatically, two processes occur: (1) the air expands and (2) some of its moisture condenses. The observed vertical molar density gradient \(\partial N_v/\partial z\) of water vapor reflects both processes. With increasing height molar density of water vapor is reduced by condensation (affecting vapor only) and by expansion (affecting all the gases). Therefore, the decrease of molar density due to condensation alone can be written as the difference \(\partial N_v/\partial z - (N_v/N)\partial N/\partial z\). The second term describes the expansion of vapor at a constant mixing ratio. If the vapor did not condense, its molar density would decrease with height as a constant proportion of the total molar density.
Thus for $S(x, z)$ (2) we have:

$$S(x, z) = w \left( \frac{\partial N_v}{\partial z} - \frac{N_v \partial N}{N \partial z} \right). \tag{3}$$

As Eq. (3) describes condensation rate in the ascending region of the flow, here $w > 0$ is vertical velocity of the ascending air.

Equation (2) can be split into two equations, one for the dry air, which is conserved, another for the saturated water vapor, which is not conserved:

$$\frac{\partial (N_d u)}{\partial x} + \frac{\partial (N_d w)}{\partial z} = 0. \tag{4}$$

$$\frac{\partial (N_v u)}{\partial x} + \frac{\partial (N_v w)}{\partial z} = w \left( \frac{\partial N_v}{\partial z} - \frac{N_v \partial N}{N \partial z} \right). \tag{5}$$

We now consider a horizontally isothermal area, such that the molar density of saturated water vapor does not depend on $x$, $\partial N_v / \partial x = 0$. Expanding the terms in Eq. (5) we have

$$N_v \frac{\partial u}{\partial x} + u \frac{\partial N_v}{\partial x} + N_v \frac{\partial w}{\partial z} + w \frac{\partial N_v}{\partial z} = w \frac{\partial N_v}{\partial z} - w \frac{N_v \partial N}{N \partial z}. \tag{6}$$

We notice that the two terms in (6), one before and another after the equal sign, cancel; we also note that the second term in the left-hand side of the equation is zero, as specified for the isothermal horizontal plane, where $\partial N_v / \partial x = 0$; we finally multiply both parts of the equation by $N/N_v$ and group terms to obtain from Eq. (5)

$$N \frac{\partial u}{\partial x} + N \frac{\partial w}{\partial z} + w \frac{\partial N}{\partial z} = 0. \tag{7}$$

Expanding the terms in Eq. (2) and using Eq. (3) we have

$$u \frac{\partial N}{\partial x} + N \frac{\partial u}{\partial x} + N \frac{\partial w}{\partial z} + w \frac{\partial N}{\partial z} = w \left( \frac{\partial N_v}{\partial z} - \frac{N_v \partial N}{N \partial z} \right). \tag{8}$$

Now using Eq. (7) and dividing both parts of the equation by $u$ we find from (8):
\[
\frac{\partial N}{\partial x} = \left(\frac{\partial N_e}{\partial z} - \frac{N_e \partial N}{N \partial z}\right) \frac{w}{u}.
\] (9)

Using the ideal gas law (1) we can express Eq. (9) in terms of pressure rather than density. Noting that \(\partial N/\partial z = (1/RT)[\partial p/\partial z - (p/T)\partial T/\partial z]\) (and a similar equation holds for \(N_v\)) we have from Eq. (9) at \(\partial T/\partial x = 0\):

\[
\frac{\partial p}{\partial x} = \left(\frac{\partial p_v}{\partial z} - \frac{p_v \partial p}{p \partial z}\right) \frac{w}{u}.
\] (10)

This is a fundamental result: the non-equilibrium vertical pressure gradient of the condensable component causes a horizontal gradient of total air pressure. The physics of Eq. (10) relates to the fact that the vertical scale height \(h_v\) of the condensable (non-conserved) component is significantly smaller than the scale height \(h\) of the mixture as a whole. Scale heights \(h_v\) of saturated water vapor and \(h\) of air pressure in hydrostatic equilibrium are derived, respectively, from the Clausius-Clapeyron equation, \(dp_v/p_v = (L/RT)dT/T\), and the hydrostatic balance equation [Weaver and Ramanathan, 1995; Curry and Webster, 1999; Makarieva and Gorshkov, 2007]:

\[
\frac{\partial p_v}{\partial z} = -\frac{p_v}{h_v}, \quad h_v \equiv \frac{RT^2}{L \Gamma}, \quad \Gamma \equiv -\frac{\partial T}{\partial z};
\] (11)

\[
\frac{\partial p}{\partial z} = -\rho g = -\frac{p}{h}, \quad h \equiv \frac{RT}{M g}.
\] (12)

Here \(L = 45\ \text{kJ mol}^{-1}\) is the molar heat of vaporization, \(\rho = NM\) is air density, \(M \approx 29\ \text{g mol}^{-1}\) is molar mass of air. We note that the scale height of saturated water vapor is inversely proportional to the lapse rate. At \(h_v < h\) the term in brackets in the right-hand side of Eq. (10) becomes \(-p_v/h_v + (p_v/p)p/h = p_v(h_v - h)/(h_v h) < 0\).

Based on these relationships we can estimate the characteristic magnitude of the horizontal pressure gradients associated with condensation. We ask what surface pressure...
gradient would be induced by condensation alone in the lower atmosphere in the tropics and how does this compare to observations? We take the following values: \( T = 300 \text{ K} \), \( p_v = 35 \text{ hPa} \), \( \Gamma = 4.5 \text{ K} \text{ km}^{-1} \) [Mapes, 2001], \( w/u = 10^{-3} \) [Rex, 1958]. Here \( u \) (meridional velocity) describes the meridional component of the Hadley circulation (the \( x \) axis is chosen perpendicular to the equator). Using these values we have from Eq. (10):

\[
\frac{\partial p}{\partial x} = -p_v \left( \frac{1}{h_v} - \frac{1}{h} \right) \frac{w}{u} = -\frac{p_v}{RT} \left( \frac{LT}{T} - Mg \right) \frac{w}{u} \approx -0.6 \text{ Pa km}^{-1}.
\] (13)

The minus sign indicates that pressure decreases in the direction of the horizontal air flow (with \( u \) directed along the \( x \) axis, \( \partial p/\partial x \) is negative). For the ascending region of a circulation of total linear size \( L \sim 2 \times 10^3 \text{ km} \) [Held and Hou, 1980] the total horizontal pressure difference will be around 6 hPa over \( 10^3 \text{ km} \), in satisfactory proximity to what is actually observed in Hadley cells [Murphree and Van den Dool, 1988].

The estimate can be obtained in a different way, by taking experimental values of the regional evaporation \( E \) and precipitation \( P \) and recalling that \( S = (E - P)/h_P \). We have from Eqs. (9) and (3):

\[
\frac{\partial p}{\partial x} = RT \frac{\partial N}{\partial x} = RT \frac{S(x, z)}{u} = RT \frac{(E - P)}{h_P} \frac{1}{u}.
\] (14)

For the characteristic value of the meridional velocity, \( u \sim 2 \text{ m s}^{-1} \) [Rex, 1958], taking \( (E - P) \sim -3 \text{ m year}^{-1} = 5 \times 10^{-3} \text{ mol H}_2\text{O m}^{-2} \text{ s}^{-1} \), \( h_P \sim 10 \text{ km} \) and \( T = 300 \text{ K} \) we obtain from (14) \( \partial p/\partial x \sim -0.6 \text{ Pa km}^{-1} \) in agreement with Eq. (13).

These consistent estimates, (13) and (14), illustrate and emphasize that our condensation-based pressure mechanism when coupled to fundamental atmospheric parameters, yields horizontal pressure gradients of magnitudes similar to those observed.
in real contexts. Such gradients have significant implications for both local and global atmospheric motion.

Having performed our analysis for an isothermal surface we have disregarded horizontal differential heating and any associated buoyancy effects. Accounting for temperature gradients would yield a more complete picture but this requires further research and analysis. Here we only note that the condensation-associated pressure reduction may override the conventional buoyancy effect and make cold air rise rather than descend, as observed in various atmospheric contexts in the tropical region [see, e.g., Folkins, 2006; Montgomery et al., 2006, Fig. 4c]. The outlined physical approach provides insights into the dynamics of such phenomena.

3. The physical meaning of condensation-induced pressure gradients

As moist air ascends adiabatically, it cools. The water vapor contained in the ascending air reaches saturation and condenses. Note that marked cooling is needed to produce condensation in the ascending air as gravitational expansion reduces vapor pressure as the air ascends and the temperature drop needs to overcome this contradictory factor. Cooling must override expansion to keep the vapor saturated. Brunt [1934] and McDonald [1964] evaluated the effect in relation to water’s vaporization constant $L$. If the vaporization constant is too low, the ascending and expanding water vapor will always remain unsaturated despite its decreasing temperature. McDonald [1964] also emphasized the importance of these relationships for atmospheric dynamics and encouraged the research community to give this attention ... but as far as we can tell this suggestion has not been followed until now. Another way to address how vapor behaves with increasing altitude is to note that condensation occurs if the lapse rate in the ascending air exceeds a critical
value [Makarieva et al., 2006; Makarieva and Gorshkov, 2007, 2009a]. This critical lapse rate is obtained by equating $h_v$ (11) and $h$ (12); it is a function of the acceleration of gravity, temperature and molar mass and is inversely proportional to $L$.

The dry air mass is conserved; consequently, it moves along closed trajectories, such that the ascending motion is accompanied by horizontal motion. As the air moves horizontally and ascends, the total mass of gas in the column is diminished by condensation. This, via rapid hydrostatic adjustment, reduces air pressure in the lower atmosphere. In the result, as one follows an air streamline at a given height along the horizontal axis, one observes a reduction of air pressure. In a nutshell, this is the physical meaning of the condensation-induced pressure gradient (10).

Our analyses highlight the positive physical feedback between the air motion and condensation-induced pressure gradient. The atmosphere in contact with the ocean which, via surface temperature, determines the surface value of water vapor partial pressure at an approximately constant relative humidity of 80% [Held and Soden, 2000], is dynamically unstable. Horizontal air motion accompanied by air ascent induces condensation, and this condensation produces pressure gradients to sustain the horizontal motion. The possibility of such a feedback has been previously recognized [e.g., Qiu et al., 1993], but the effect has not been thoroughly investigated.

So what starts such a condensation-driven process of air movement? Does the condensation or motion come first? Consider a motionless atmosphere with water vapor saturated at the surface. Any occasional adiabatic displacement of an air parcel upwards results in its cooling, which produces condensation, diminishes the total amount of gas in the column and drives further motion. This is sustained as long as the incoming air
is saturated with water vapor. Thus, instability underlies the gradient of water vapor partial pressure and turns the latter into a store of potential energy available for driving atmospheric movement [Makarieva et al., 2006; Makarieva and Gorshkov, 2007, 2009a].

One reviewer has challenged us to better consider the mechanisms governing atmospheric circulation as these modify atmospheric pressure gradients in the balanced flow. Indeed, in a stationary balanced flow like that of Hadley circulation the pressure gradient force acting on the air is balanced by other forces, including those associated with turbulent friction and rotation. In the simplest case of geostrophic balance the pressure gradient force is balanced by the velocity-dependent Coriolis force. This means that if one knows the pressure gradient force, one can calculate wind velocity, and vice versa. For example, Murphree and Van den Dool [1988] used the observed tropical pressure gradients in the equations of hydrodynamics to derive the observable wind speeds. Importantly, the magnitude of the turbulent friction force and Coriolis force depend on air velocity and disappear if the velocity is zero. Thus, these forces do not act on motionless air and cannot make it move. In contrast, the pressure gradient force acting on air makes it move and accumulate (or sustain) the kinetic energy that otherwise continuously dissipates. This initial motion leads to the appearance of the velocity-dependent forces which, in turn, can modify the pressure field and the resulting pattern of the balanced flow. It is in this sense that a physical process that generates pressure gradients is the primary driver of circulation. In the absence of such a process the momentum balance of the stationary flow would be destroyed by frictional dissipation. As is clear from our estimates (13, 14), the condensation-induced pressure gradients are of sufficient magnitude for the dynamic effects of condensation to play a major role in sustaining the observed Hadley circulation.
4. Discussion

Here we wish to draw the attention of climate theorists to what we consider a major and undeservingly neglected physical mechanism that, as we propose, may influence and perhaps govern many aspects of atmospheric dynamics. In our approach we focus on the driving force behind, rather than the pattern of, atmospheric motion. Our proposal is that the atmosphere moves because of condensation – under our theory it is condensation which produces pressure gradients that are translated into winds. We start by noting that if there was no driving force the atmosphere would be static. We seek to clarify the driving forces behind the existing circulation. If the pressure gradients we identify occur in nature they will by necessity produce atmospheric movement. Indeed, according to Newton’s law as embedded into the equations of hydrodynamics, the motion occurs only because a pressure gradient force has acted on the atmospheric air.

We note that currently in the atmospheric science there is both a place and a need for new approaches. Modern global circulation models do not satisfactorily account for the water cycle of the Amazon river basin, with the estimated moisture convergence being half the actual amounts estimated from the observed runoff values [Marengo, 2004]. Major problems have been identified with the prevailing thermodynamic approach to describing hurricanes [Smith et al., 2008; Makarieva et al., 2010]. Furthermore, so far it has not been possible to derive a quantitatively realistic theory of Hadley circulation based on the effects of differential heating alone [Held and Hou, 1980; Fang and Tung, 1999; Schneider, 2006]. With efforts to address this challenge ongoing [e.g., Lindzen and Hou, 1988; Robinson, 2006; Walker and Schneider, 2005, 2006], in a recent review Schneider [2006] admitted that for a dry atmosphere such a theory just hopefully remains within
reach. The problems of addressing the role of atmospheric moisture and, particularly, lack of relevant theoretical concepts, were identified as a persistent challenge. Meanwhile the incomplete understanding of the general circulation in the research literature precludes a theory-based analysis, from fundamental physical principles, of the role of latitudinal atmospheric mixing in stabilizing the Earth’s thermal regime important: a key issue in debates concerning climate sensitivity [e.g., Lindzen and Choi, 2009; Trenberth et al., 2010]. Remarkably, all these challenges concern atmospheric movements with a conspicuous potential to be influenced by water vapor. We believe that condensation-induced dynamics will yield meaningful insights into these and many other important issues.

The basic physical mechanisms underlying the driving force behind atmospheric motion and their potential magnitudes are of fundamental significance. Atmospheric theorists have tended to ascribe atmospheric movement to temperature gradients and buoyancy-driven convection - while such mechanisms appear widely accepted, many essential issues, as illustrated briefly above, remain unresolved. Here we offer a new and credible alternative mechanism for a rigorous scrutiny by the research community. It will be fascinating to see how the outlined physical mechanism can be incorporated in integrated picture of atmospheric motion. We foresee some years of fruitful advances based on studying the air motion associated with the phase transitions of water both theoretically and empirically [see, e.g., Chikoore and Jury, 2010].

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