Horizontal atmospheric pressure gradients associated with condensation of water vapor

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Abstract. Condensation of water vapor in adiabatically ascending air produces a non-equilibrium vertical gradient of water vapor pressure. Here we show, based on an analysis of the continuity equation for a compressible gas mixture of condensable and non-condensable gas components, that the loss of mass of the condensable gas (water vapor) through condensation results in the formation of a horizontal gradient of total air pressure. The magnitude of this gradient is roughly proportional to the vertical non-equilibrium pressure gradient of the condensable component multiplied by the ratio of vertical to horizontal velocity. For large scale circulation features our estimated condensation-induced horizontal pressure gradient appears close to observations, at around 0.6 Pa km^{-1} for Hadley cells. We conclude that condensation-induced dynamics based on vapor non-conservation merits further attention as a driver of atmospheric motions on Earth.

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1. Introduction

It has been noted in a number of studies that precipitation reduces local air pressure [*Lorenz*, 1967; *Trenberth et al.*, 1987; *Trenberth*, 1991; *Gu and Qian*, 1991; *Ooyama*, 2001; *Schubert et al.*, 2001; *Qiu et al.*, 1993; *Lackmann and Yablonsky*, 2004]. This conclusion is based on the physical observation that in hydrostatic equilibrium air pressure is equal to the weight of gas in the air column of a unit area. Hence, removal of substance via precipitation leads to pressure reduction in the air column. However, no comprehensive attempt has been made to describe the spatial pressure changes associated with condensation from basic principles. Here we perform such an analysis to show that condensation of water vapor produces horizontal pressure gradients of observable magnitudes and thus represents a significant driver of atmospheric dynamics on Earth.

We consider the atmosphere as a mixture of a non-condensable component (dry air) with molar density N_d and a condensable component (water vapor) with molar density N_v. Molar density N of moist air is thus $N = N_d + N_v$. The ideal gas equation of state relates pressure p to temperature T and molar density N via the universal molar gas constant $R = 8.3$ J mol^{-1} K^{-1} independently of molar mass:

$$p = NRT, \quad p_v = N_vRT, \quad p_d = N_dRT. \tag{1}$$

Here p, N, p_v, N_v, p_d, N_d are the pressure and molar density of moist air as a whole, water vapor and dry air, respectively.

To provide conceptual transparency we consider the case of a stationary two-dimensional circulation pattern. The continuity equation for a gas mixture, where some component (here water vapor) is not conserved, then reads:

$$\frac{\partial(Nu)}{\partial x} + \frac{\partial(Nw)}{\partial z} = S(x,z). \tag{2}$$

Here u is horizontal velocity, w is vertical velocity and $S(x,z)$ is the volume-specific rate of "loss" (condensation) of the condensable component (mol m^{-3} s^{-1}).

In the atmosphere the sink term S is equal to the difference between volume-specific evaporation rate $\tilde{E}$ and condensation rate $\tilde{C}$, $S = \tilde{E} - \tilde{C}$. Local recycling (evaporation – condensation – re-evaporation) does not change the amount of vapor in the local volume, but their difference does. Since we consider a stationary case where local concentrations of all components are constant, the difference $\tilde{E} - \tilde{C}$ is equal to the amount of local volume-specific precipitation, i.e., to the rate $-\tilde{P} < 0$ of removal of liquid water from the local volume, $S = \tilde{E} - \tilde{C} = -\tilde{P}$ [e.g., *Sabato*, 2008].

2. Derivation of the Horizontal Pressure Gradient

As the air ascends, two processes occur: (1) the air expands and (2) some of its moisture condenses. The observed vertical profile of the molar density gradient $\partial N_v/\partial z$ of water vapor reflects both processes. Molar density of the water vapor is reduced by condensation and by expansion as the other non-condensable gases in equilibrium. The decrease of molar density due to condensation alone can be written as $\partial N_v/\partial z - (N_v/N)\partial N/\partial z$. The second term describes the expansion of vapor at a constant mixing ratio. If the vapor did not condense, its molar density would decrease with height as a constant proportion of the total molar density. Thus for $S(x,z)$ (2) we have:

$$S(x,z) = w\left(\frac{\partial N_v}{\partial z} - \frac{N_v}{N}\frac{\partial N}{\partial z}\right). \tag{3}$$

Equation (2) can be split into two equations, one for the dry air, which is conserved, another for the saturated water vapor, which is not conserved:

$$\frac{\partial(N_d u)}{\partial x} + \frac{\partial(N_d w)}{\partial z} = 0. \tag{4}$$

$$\frac{\partial(N_v u)}{\partial x} + \frac{\partial(N_v w)}{\partial z} = w\left(\frac{\partial N_v}{\partial z} - \frac{N_v}{N}\frac{\partial N}{\partial z}\right). \tag{5}$$

We consider a horizontally isothermal area, such that the saturated partial of water vapor does not depend on x, $\partial N_v/\partial x = 0$. Expanding the terms in Eq. (5) we have

$$N_v\frac{\partial u}{\partial x} + u\frac{\partial N_v}{\partial x} + N_v\frac{\partial w}{\partial z} + w\frac{\partial N_v}{\partial z} = w\frac{\partial N_v}{\partial z} - w\frac{N_v}{N}\frac{\partial N}{\partial z}. \tag{6}$$

We notice that the two terms in (6), one before and another after the equal sign, cancel; we also note that the second term in the left-hand side of the equation, $u(\partial N_v/\partial x)$, is zero, as specified for the isothermal horizontal plane, where $\partial N_v/\partial x = 0$; we finally multiply both parts of the equation by N/N_v and group all terms in one side of the equation to obtain from Eq. (5)

$$N\frac{\partial u}{\partial x} + N\frac{\partial w}{\partial z} + w\frac{\partial N}{\partial z} = 0. \tag{7}$$

Expanding the terms in Eq. (2) and using Eq. (3) we have

$$u\frac{\partial N}{\partial x} + N\frac{\partial u}{\partial x} + N\frac{\partial w}{\partial z} + w\frac{\partial N}{\partial z} = w\left(\frac{\partial N_v}{\partial z} - \frac{N_v}{N}\frac{\partial N}{\partial z}\right). \tag{8}$$

Now using Eq. (7) and dividing both parts of the equation by u we find:

$$\frac{\partial N}{\partial x} = \left(\frac{\partial N_v}{\partial z} - \frac{N_v}{N}\frac{\partial N}{\partial z}\right)\frac{w}{u}. \tag{9}$$

Using the ideal gas law (1) we can express Eq. (9) in terms of pressure rather than density. Noting that $\partial N/\partial z = (1/RT)[\partial p/\partial z - (p/T)\partial T/\partial z]$ (and a similar equation holds for N_v) we have from Eq. (9) at $\partial T/\partial x = 0$:

$$\frac{\partial p}{\partial x} = \left(\frac{\partial p_v}{\partial z} - \frac{p_v}{p}\frac{\partial p}{\partial z}\right)\frac{w}{u}. \tag{10}$$

The terms containing vertical temperature gradient $\partial T/\partial z$ that appeared after differentiating $\partial N/\partial z$ and $\partial N_v/\partial z$ have cancelled each other, so the resulting expression relates pressure changes only.

This is a fundamental result: the non-equilibrium vertical pressure gradient of the condensable component leads to the appearance of a horizontal gradient of total air pressure. The physics of Eq. (10) consists in the fact that the vertical scale height h_v of the condensable (non-conserved) component is significantly smaller than the scale height h of the mixture as a whole. Scale heights h_v of saturated water vapor and h of air pressure in hydrostatic equilibrium are derived, respectively, from the Clausius-Clapeyron equation, $dp_v/p_v = (L/RT)dT/T$, and the hydrostatic balance equation [*Weaver and Ramanathan*, 1995; *Curry and Webster*, 1999; *Makarieva and Gorshkov*, 2007]:

$$\frac{\partial p_v}{\partial z} = -\frac{p_v}{h_v},\ h_v \equiv \frac{RT^2}{L\Gamma},\ \Gamma \equiv -\frac{\partial T}{\partial z}; \tag{11}$$

$$\frac{\partial p}{\partial z} = -\rho g = -\frac{p}{h},\ h \equiv \frac{RT}{Mg}. \tag{12}$$

Here $L = 45$ kJ mol^{-1} is the molar heat of vaporization, $\rho = NM$ is air density, $M \approx 29$ g mol^{-1} is molar mass of air. We note that the scale height of saturated water vapor is inversely proportional to the lapse rate. At $h_v < h$ the term in brackets in the right-hand side of Eq. (10) becomes $-p_v/h_v + (p_v/p)p/h = p_v(h_v - h)/(h_v h) < 0$.

The characteristic magnitude of the horizontal pressure gradient associated with condensation can be estimated on the example of the meridional component of the Hadley circulation – an approximately parallel flow converging in the area of ascent. We ask: what surface pressure gradient would be induced by condensation alone in the lower atmosphere in the tropics? We take the following values: $T = 300$ K, $p_v = 35$ hPa, $\Gamma = 4.5$ K km^{-1} [*Mapes*, 2001], $w/u = 10^{-3}$ [*Rex*, 1958]. Using these values we have from Eq. (10):

$$\frac{\partial p}{\partial x} = -p_v \left(\frac{1}{h_v} - \frac{1}{h}\right)\frac{w}{u} = -\frac{p_v}{RT}\left(\frac{L\Gamma}{T} - Mg\right)\frac{w}{u} \approx -0.6\,\mathrm{Pa\,km^{-1}}. \qquad (13)$$

The minus sign indicates that pressure decreases in the direction of the horizontal air flow (with u directed along the x axis, $\partial p/\partial x$ is negative). For a circulation of linear size $L \sim 2 \times 10^3$ km [*Held and Hou*, 1980] the total horizontal pressure difference will be in the order of 12 hPa, in satisfactory proximity to what is actually observed in Hadley cells [*Murphree and Van den Dool*, 1988]. The obtained above first-order estimate (13) derived with use of characteristic atmospheric parameters is meant to emphasize the importance of the condensation-induced pressure gradients rather than to accurately determine their actual value. As is clear from (13), these gradients are of a sufficient magnitude to play a major role in sustaining the observed Hadley circulation.

Note that having performed our analysis for an isothermal surface we have disregarded the horizontal differential heating and the associated buoyancy effects. Introducing temperature gradients into the outlined physical picture will require further research and analysis. In the meantime, we note that in various atmospheric contexts involving precipitation evidence is accumulating that the condensation-associated pressure reduction can override the conventional buoyancy effect (i.e., that the warm air rises) and make *cold* air rise rather than descend. Indeed, recent studies of temperature profiles in intense hurricanes indicate that the air that ascends violently at the eyewall is conspicuously colder than the air descending within the eye as well as outside the hurricane area [see, e.g. *Montgomery et al.*, 2006, Fig. 4c]. The outlined physical approach provides clues to understanding these and similar phenomena [e.g., *Folkins*, 2006].

3. The physical meaning of condensation-induced pressure gradients

As the moist air ascends, it cools. The water vapor the ascending air contains reaches saturation and condenses. It is important to bear in mind that marked cooling is needed to produce condensation in the ascending air. Indeed, the gravitational expansion reduces the water vapor pressure as the air ascends and would drive it away from saturation. The cooling must be significant enough to override the effect of expansion and keep the vapor saturated. *Brunt* [1934] and *McDonald* [1964] evaluated the effect in terms of the stipulation it sets upon the vaporization constant L of water vapor. If the vaporization constant is too low, the ascending and expanding water vapor will always remain unsaturated despite its decreasing temperature. (Notably, *McDonald* [1964] emphasized the crucial importance of these relationships for understanding atmospheric dynamics and strongly encouraged the research community to give this attention. But as far as we can tell this suggestion has not been followed.) The same problem can be approached differently: condensation occurs if the lapse rate in the ascending air exceeds a certain critical value [*Makarieva et al.*, 2006; *Makarieva and Gorshkov*, 2007, 2009a]. This critical lapse rate is obtained by equating h_v (11) and h (12); it is a function the acceleration of gravity, temperature and molar mass and is inversely proportional to L.

The dry air mass is conserved; consequently, the air moves along closed trajectories. The ascending motion is invariably accompanied by horizontal motion. As the air moves horizontally and ascends, the water vapor condenses. The total mass of gas in the column diminishes. This, via rapid hydrostatic adjustment, leads to the reduction of air pressure in the lower atmosphere. Therefore, as one approaches towards the convergence area at a given height, the air pressure falls. In a nutshell, this is the physical meaning of the condensation-induced pressure gradient (10).

Our main result, Eq. (10), highlights the positive physical feedback between the air motion and condensation-induced pressure gradient. The atmosphere in contact with the ocean which, via surface temperature, determines the surface value of water vapor partial pressure at roughly constant relative humidity of 80% [*Held and Soden*, 2000], is dynamically unstable. The converging air motion is accompanied by air ascent that induces condensation, and condensation produces pressure gradients to sustain the converging motion. The possibility of such a feedback has been previously recognized [e.g., *Qiu et al.*, 1993], but the effect has not been thoroughly investigated. What comes first, condensation or motion? Consider a motionless atmosphere with water vapor saturated at the surface. An occasional displacement of an air parcel upwards results in its cooling, which produces condensation, diminishes the total amount of gas in the column and initiates the converging air motion. This motion is sustained as long as the incoming air is saturated with water vapor. Thus, the condensational instability is the primary cause that turns the partial pressure of water vapor into a store of potential energy available for conversion to the kinetic energy of air masses upon condensation [*Makarieva et al.*, 2006; *Makarieva and Gorshkov*, 2007, 2009a].

4. Discussion

Here we wish to draw the attention of climate theorists to what we consider a major and undeservingly neglected circulation driver. The condensation-induced pressure gradients that we have been examining are associated with the horizontal gradients of air density $\rho = NM$ that have been conventionally considered as minor and thus ignored in the continuity equation [e.g., *Sabato*, 2008]. For example, a typical $\Delta p = 50$ hPa pressure difference observed along the approximately horizontally isothermal surface between the outer environment and the hurricane center [e.g., *Holland*, 1980] is associated with a density difference of only around 5%. This density difference can be safely neglected when estimating the resulting air velocity u from the *known* pressure differences Δp. Here the basic scale relation is given by Bernoulli's equation, $\rho u^2/2 = \Delta p$. The point is that a 5% change in ρ does not significantly impact the magnitude of the estimated air velocity *at a given* Δp. But, as we have shown, for the determination of the pressure gradient (10) the density gradient (9) is key.

Indeed, considering the equation of state (1) for the horizontally isothermal surface we have $p = C\rho$, where $C \equiv RT/M =$ const. Irrespective of why the considered pressure difference arises, from Bernoulli's equation we know that $u^2 = 2\Delta p/\rho = 2C\Delta\rho/\rho$, $\Delta\rho = \rho_0 - \rho$. Thus, if one puts $\Delta\rho/\rho = \Delta p/p$ equal to zero, no velocity forms and there is no circulation. Indeed, we have $u^2 = 2\Delta p/\rho = 2C\Delta\rho/\rho = 2C(\Delta\rho/\rho_0)(1 + \Delta\rho/\rho_0 + ...)$. As one can see, discarding $\Delta\rho$ compared to ρ does indeed correspond to discarding the higher order term of the smallness parameter $\Delta\rho/\rho$. But with respect to the pressure gradient, the main effect is proportional to the smallness parameter $\Delta\rho/\rho_0$ itself. If the latter is assumed to be zero, the effect is overlooked. We suggest that this dual aspect of the magnitude of condensation-related density changes has not been recognized and this has contributed to the neglect of condensation-associated pressure gradients in the Earth's atmosphere.

We note that currently in the atmospheric science there is both a place and a need for new approaches. Modern global circulation models do not satisfactorily account for the water cycle of the Amazon river basin, with the estimated moisture convergence being half the actual amounts estimated from the observed runoff values [*Marengo*, 2004]. So far it has not been possible to derive a quantitatively realistic theory of Hadley circulation based on the effects of differential heating alone [*Held and Hou*, 1980; *Fang and Tung*, 1999; *Schneider*, 2006]. This does not allow one to analyze in theory, based on fundamental physical principles, the role of latitudinal atmospheric mixing in stabilizing the thermal regime of the planet, one of the important issues in the climate sensitivity debates [e.g., *Lindzen and Choi*, 2009; *Trenberth et al.*, 2010]. The prevailing thermodynamic approach to hurricane description has been shown to suffer from major physical inconsistencies [*Smith et al.*, 2008; *Makarieva et al.*, 2010]. Remarkably, all these problems concern circulation patterns with a conspicuous role of the phase transitions of water vapor. It is our assertion that condensation-induced dynamics can yield meaningful insights into these and hopefully many other important problems. An exciting new field of work awaits interested researchers.

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